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UNSTEADY HEAT TRANSFER BETWEEN A FLUID, WITH TIME VARYING TEMPERATURE, AND A PLATE: AN EXACT SOLUTION

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Abstract—An analysis using Laplace transformations is made for the transient temperature distribution and surface heat flux when a plate which is convectively cooled from below has a fluid passing over it whose free stream temperature at the plate leading edge varies arbitrarily with time, and when the plate's thermal response is coupled to the fluid via the conjugation conditions at the interface.

First the solution for a step function is found and then generalized to handle arbitrary fluid temperature variation with time. A simple to use approximate method is presented for the most general case. For comparison, quasi-steady results are also derived.

NOMENCLATURE

b, thickness of plate;
C_p, specific heat;
erf, error function;
$F, F_i^{r'}, F_i^r, F_i^s$, surface heat flux functions defined by equations (23) through (26);
h_c, heat-transfer coefficient between bottom of plate and coolant;
i, index;
$i^1\,\text{erfc}[z]$, first repeated integral of the error function $= \int_z^\infty \text{erfc}[v]\,dv$;
j, index;
k_f, thermal conductivity of fluid moving over top of plate;
p, Laplace transform parameter;
Q_w, nondimensional surface heat flux defined by equation (11);
q_w, surface heat flux;
r, $= \rho_w C_{p,w} b/k_f$;
s, Laplace transform parameter;
$T(x, y, t)$, temperature;
T_c, coolant temperature;
T_i^-, T_i^+, temperature just before and after, respectively, a step change in temperature at time t_i;
T_0, fluid temperature at $x = 0$ in the ultimate steady state;
$T_{x=0}$, instantaneous fluid temperature at $x = 0$;
t, time;
u_∞, free stream velocity;
$u(\tau-1)$, unit step function, equals 0 for $\tau < 1$, and $+1$ for $\tau \geqslant 1$;
x, space coordinate along plate;
Y, $\frac{y}{2}\sqrt{(u_\infty/\alpha_f x)}$ nondimensional y coordinate;
y, space coordinate perpendicular to plate.

Greek symbols

α_f, thermal diffusivity of fluid;
δ_t, thermal boundary-layer thickness;
ε, $= \sqrt{(x/\alpha_f u_\infty)}/2r$, coupling parameter;
η, $= \frac{h_c}{k_f}\sqrt{(\alpha_f x/u_\infty)}$;
θ, $= T - T_0$, temperature excess;
λ, dummy variable for τ;
ξ, dummy variable for x;
ρ, mass density;
σ_w, $= T_w - T_c$;
$\Delta\sigma_{w_i}^{r'}, \Delta\sigma_{w_i}^r, \Delta\sigma_{w_i}^s$, temperature functions defined by equations (20) to (22);
$\sigma_{x=0}$, $= T_{x=0} - T_c$;
τ, $= u_\infty t/x$, nondimensional time;
τ_1, nondimensional time at end of linearly increasing fluid temperature;
ϕ, $= \tau - 1$, shifted nondimensional time.

Subscripts

c, coolant conditions below plate;
f, properties of fluid passing over the plate;
q.s., quasi-steady conditions;
w, plate condition.

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INTRODUCTION

THIS study of the transient temperature distribution and surface heat flux, between a plate cooled from below and interacting with a fluid passing over it with a time varying temperature at the plate leading edge, was instigated by the transient induced in a gas turbine blade or vane when the turbine inlet temperature changes because of start-up or shut-down or simply as a result of a change in power level when operating. The results are also thought applicable to other components of both jet and rocket engines, to portions of liquid metal cooled nuclear reactors, and to heat exchangers, the transient response of a recuperative exchanger, and the transient and periodic unsteady response of a regenerative exchanger.

Many previous investigations [1–12] attack the problem of the transient heat transfer initiated by a step change in the wall temperature or some other controlled change in the temperature of a surface over which a fluid is passing. Characteristic of the step change in the wall temperature is the large surface heat fluxes attained in the one-dimensional conduction phase of the problem which occurs before any of the fluid which was at $x = 0$ at $t = 0$ reaches any point of interest on the plate. Solutions for step changes in the plate surface heat flux have also been obtained in many of these investigations. Additionally, in all these works, the plate boundary condition used was a pure function of time and, hence, did not vary with position on the plate, and the velocity field was steady.

Reference [13] among others, treats the transient heat transfer process induced by a time varying velocity field.

Reference [14], in connection with the dynamic response of heat exchangers, uses a quasi-steady analysis and Laplace transforms to solve the transient, in a circular tube and the fluid flowing through it, caused by time dependent generation in the tube wall.

Chambre [15] used Laplace transformations to solve for the heat-transfer response of a plate, insulated on its bottom surface, with a fluid passing over its upper surface with a slug flow velocity profile, and the plate containing time dependent heat sources which are turned on at time $t = 0$. Proper consideration was given to the mutual coupling between the temperature fields of the fluid and plate by utilizing the conjugation conditions, continuity of temperature and heat flux, at the interface between the plate and the fluid.

In the references mentioned so far, the transient convection heat transfer is caused by either a controlled change in the thermal condition of the solid or a controlled change in the velocity field as opposed to prescribed changes in the fluid temperature field. Konopliv and Sparrow [16] use Laplace transforms and series methods to deal with the case of Stokesian flow about a sphere when the fluid environment temperature changes abruptly. Their analysis handles the situation of constant sphere surface temperature and also the conjugate problem where the temperature of the entire sphere is lumped in the space coordinates and can vary with time as a result of its interaction with the fluid. Inouye and Yoshinaga [17] use an approximate integral method due to Liepmann on two problems of transient heat transfer at a stagnation point due to a sudden change in free stream temperature. Their first problem considers an isothermal surface suddenly subjected to a free stream temperature change. Because all of the fluid reaches the new temperature instantly, this is the same as the problem of abruptly changing the surface temperature at a stagnation point which was dealt with earlier by Sparrow [1] and Chao and Jeng [8]. Their second problem, however, considers a thin flat plate with its rear surface insulated when the temperature of the stagnation flow is abruptly raised. Now the problem is a conjugate one and their approximate integral solution yields plate temperature and heat-transfer coefficient as a function of time and a parameter which is, basically, a measure of the plate's thermal capacity. They remark that their solution and a quasi-steady solution are very close for typical plate fluid combinations, such as an iron plate and air. Lyman [18] analyzes transient heat transfer at a stagnation point due to an abrupt change in the free stream temperature at the edge of a thermal boundary layer when the solid forming the stagnation point is either semi-infinite, or of a finite thickness with an insulated lower surface. Until the thermal disturbance propagates from the outer edge of the thermal boundary layer to the solid, the solid does not participate in the energy transfer process. An important result is the dimensionless transit time, the time needed before the wall feels the change in free stream temperature, as a function of the thermal boundary-layer thickness. For times greater than this, the conjugate problem involving the coupled response of the wall material and the fluid is solved for the semi-infinite wall and then, by using small time and large time solutions, for the finite thickness wall insulated on its lower surface. One of the interesting observations of this solution is the absence of the infinite, and very large values of heat flux at short times which occur when the wall temperature is changed abruptly. The reason for this, of course, is the transit time required before the solid can feel the effect of the abrupt change of the free stream temperature which occurred at the edge of the thermal boundary layer. This time allows the temperature profiles in the fluid to become smooth and continuous by the time any of the heated fluid reaches the solid wall. Lyman also presents the appropriate quasi-steady solutions and notes that when the fluid is air and the solid has thermal properties of glass or aluminum, the simpler quasi-steady solution suffices. Landram [19] works the problem of wall-to-fluid heat transfer in turbulent tube flow with an inlet temperature prescribed as a function of time. His analysis, using Laplace transforms, employs a constant quasi-steady heat-transfer coefficient, and the results are for tube walls which are semi-infinite in thickness. Hence, it is a short time analysis for finite thickness tube walls. Graphs of response curves are presented for the case of a step change in the inlet temperature. In [20] Zargary and Brock derive an integral equation for the fluid temperature at the interface, when fluid is flowing through a pipe whose outside walls are insulated and a transient is initiated by virtue of a step change in the fluid inlet temperature. However, no solutions to the integral equation are presented. Sparrow and DeFarias [21] solve, in an exact fashion, the unsteady problem of a fluid flowing, in a steady laminar slug flow fashion, between two parallel plates with insulated outer surfaces when the fluid inlet temperature varies sinusoidally with time. When axial conduction is neglected in the plate and its temperature is lumped in the transverse direction, the energy balance for the plate in this problem becomes a boundary condition for the fluid. For the ultimate periodic unsteady state (initial condition has been "forgotten"), they find an exact analytical solution for the temperature, and the local Nusselt numbers. The results are compared to quasi-steady solutions employing a time independent heat-transfer coefficient and criteria are evolved for the range of validity of the quasi-steady solution.

The present work concerns itself with the analytical prediction of the transient heat transfer between a fluid, flowing in a laminar fashion, and the plate over which it flows. The plate's lower surface is exposed to a coolant at known temperature, T_c, and with a known surface coefficient of heat transfer, h_c, between the coolant and the lower plate surface. Hence, the insulated lower surface case is included since it corresponds to the degenerate situation where $h_c = 0$. Changes in the fluid temperature with time at the plate leading edge give rise to the transient in the fluid and the plate. Since the plate temperature is not specified, *a priori*, the transient temperature distribution in the plate and in the fluid passing over it are mutually coupled and this results in a conjugate problem. The governing equations are then solved by two successive Laplace transformations for the case of a step change in the fluid temperature at the leading edge. Use of Duhamel's Theorem generalizes this result for arbitrary time varying fluid temperature. A solution is then found for the case of linear fluid temperature variation with time and it is shown how this solution and the step function solution can be used to approximate the response, to arbitrary temperature variation of the fluid, to any degree of accuracy using only tabulated elementary functions; namely, the error function and the first repeated integral of the error function. Quasi-steady solutions, not employing the assumption of a time independent heat-transfer coefficient, are also obtained and compared to the exact solutions.

ANALYSIS

The physical situation is a flat plate of thickness b which has its lower surface exposed to a coolant at constant temperature T_c with surface coefficient of heat transfer h_c and the fluid flow over the top of the plate is steady and laminar. The following idealizations are made to effect an exact analytical solution. The fluid flow is low speed, constant property, and of the two-dimensional planar boundary layer type with a slug flow velocity profile. In addition, the thermal properties of the plate are constant, the plate temperature is lumped in the y coordinate direction, and axial conduction neglected. First the fundamental solution for a step change in fluid temperature at $x = 0$ is sought. Initially, the plate and fluid are both at the coolant temperature T_c, when suddenly the fluid temperature at $x = 0$, $T_{x=0}$, is changed to the value T_0 and subsequently held constant. Defining $\theta = T - T_0$, the governing partial differential equation and boundary conditions for the fluid temperature distribution become,

$$\frac{\partial \theta}{\partial t} + u_\infty \frac{\partial \theta}{\partial x} = \alpha_f \frac{\partial^2 \theta}{\partial y^2} \tag{1}$$

$$t = 0, \quad x > 0, \quad y > 0, \quad \theta = \theta_c = T_c - T_0 \tag{2}$$

$$x = 0, \quad t > 0, \quad y > 0, \quad \theta = 0 \tag{3}$$

$$y \to \infty, \quad t > 0, \quad x > 0, \quad \theta \text{ is finite.} \tag{4}$$

Now apply the energy balance to a control volume of plate b by dx and noting that the plate temperature $T_w(x, t)$ must equal the fluid temperature at $y = 0$ because of the assumption of a lumped plate temperature, the equation for the plate becomes a boundary condition on the fluid; namely,

$$y = 0, \quad t > 0, \quad x > 0, \quad r\frac{\partial \theta}{\partial t} + \frac{h_c}{k_f}(\theta - \theta_c) = \frac{\partial \theta}{\partial y} \tag{5}$$

where $r = \rho_w C_{p,w} b / k_f$. To solve equation (1) and its associated side conditions (2) through (5), a Laplace transform with respect to time t and then with respect to x was applied to equations (1)–(5). So using

$$\bar{\theta} = \int_0^\infty \theta\, e^{-pt}\, dt$$

and

$$\bar{\bar{\theta}} = \int_0^\infty \bar{\theta}\, e^{-sx}\, dx$$

equations (1) to (5) become

$$\frac{d^2\bar{\bar{\theta}}}{dy^2} - \left(\frac{u_\infty s + p}{\alpha_f}\right)\bar{\bar{\theta}} = -\frac{\theta_c}{s\alpha_f} \quad (6)$$

$$y \to \infty, \quad \bar{\bar{\theta}} \text{ is finite} \quad (7)$$

$$y = 0, \quad \frac{d\bar{\bar{\theta}}}{dy} = \left(\frac{h_c}{k_f} + rp\right)\bar{\bar{\theta}} - \theta_c\left(\frac{r}{s} + \frac{h_c}{spk_f}\right). \quad (8)$$

The details of the solution of equation (6) subject to (7) and (8) and the inverse transformation of the result, first with respect to s, and then with respect to p is given in [22]. Only the result is presented here employing the following nondimensional variables.

$$\tau = u_\infty t/x$$

$$Y = \frac{y}{2}\sqrt{(u_\infty/\alpha_f x)}$$

$$\eta = \frac{h_c}{k_f}\sqrt{(\alpha_f x/u_\infty)}$$

$$\varepsilon = \sqrt{(x/\alpha_f u_\infty)}/2r.$$

Thus the exact analytical solution for the nondimensional fluid temperature is,

$$\frac{\theta}{\theta_c} = 1 - u(\tau-1)[\text{erf}[Y] + e^{2\eta Y + \eta^2}\{\text{erf}[\varepsilon(\tau-1)+\eta+Y] - \text{erf}[\eta+Y]\}] \quad (9)$$

where

$$u(\tau-1) = \begin{cases} 0 & \text{for} \quad \tau < 1 \\ +1 & \text{for} \quad \tau \geqslant 1. \end{cases}$$

The temperature response of the plate is found by setting $Y = 0$ in equation (9), giving

$$\frac{\theta_w}{\theta_c} = 1 - u(\tau-1)[e^{\eta^2}\{\text{erf}[\varepsilon(\tau-1)+\eta] - \text{erf}[\eta]\}]. \quad (10)$$

The local surface heat flux q_w is put into nondimensional form by dividing it by the surface heat flux for an isothermal plate at θ_c under the same conditions, see [23].

$$Q_w = q_w/k_f\theta_c\sqrt{(u_\infty/\pi\alpha_f x)}. \quad (11)$$

It can be easily seen that

$$Q_w = -\frac{\sqrt{(\pi)}}{2\theta_c}\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0}.$$

Performing the indicated operations on equation (9) yields the nondimensional surface heat flux.

$$Q_w = 0 \quad \text{for} \quad \tau < 1$$

$$Q_w = \exp\{-[\varepsilon^2(\tau-1)^2 + 2\eta\varepsilon(\tau-1)]\} + \sqrt{(\pi)}\eta\, e^{\eta^2}\{\text{erf}[\varepsilon(\tau-1)+\eta] - \text{erf}[\eta]\} \quad \text{for} \quad \tau \geqslant 1. \quad (12)$$

The response functions, equations (9), (10) and (12) were checked in a number of ways. First, instead of directly inverting from the second transformed plane and then operating on the result in the physical (x, y, t) plane to get equations (10) and (12), the wall temperature and heat flux were formed in the second transformed plane (p, s, y) and the results inverted giving equations (10) and (12), respectively, as it should. Secondly, it was verified that equation (9) satisfied equations (1) through (5) by direct substitution. Thirdly, some limiting cases were examined to insure agreement with physical reasoning. As $h_c \to \infty$, $\eta \to \infty$ and this means, physically, that $T_w \to T_c$ and hence, $\theta_w \to \theta_c$ for all x and t. In addition Q_w should approach unity. Letting $\eta \to \infty$ and utilizing L'Hospital's Rule where necessary in equations (10) and (12) yields $\theta_w \to \theta_c$ and $Q_w \to 1$ as required. It is noted that r is a measure of the heat capacity of the plate. Hence, an infinite heat capacity plate (which would stay, therefore, at θ_c during a transient) corresponds to $r \to \infty$ which means that the coupling parameter $\varepsilon \to 0$. Letting $\varepsilon \to 0$ in equations (10) and (12), it is clear that $\theta_w \to \theta_c$ and $Q_w \to 1$ as they should.

Arbitrary fluid temperature variation at $x = 0$

Possession of the fundamental solution for the temperature field in the moving fluid, equation (9), due to a step change in fluid temperature at $x = 0$, together with the linearity of equation (1) allows determination of the temperature function for arbitrary inlet temperature variation via Duhamel's Theorem [24]. Although the entire temperature field can be found, it is usually only necessary to know the wall temperature and the surface heat flux. In addition, some compactness of form may be realized by defining a shifted nondimensional time ϕ as,

$$\phi = \tau - 1. \quad (13)$$

Also defining for convenience,

$$\sigma_w = T_w - T_c \quad (14)$$

$$\sigma_{x=0} = T_{x=0}(\tau) - T_c. \quad (15)$$

Duhamel's Theorem gives, for arbitrary variations of fluid temperature at $x = 0$, as the solution for the wall temperature excess,

$$\sigma_w = e^{\eta^2} \int_0^{\phi} \{\mathrm{erf}[\varepsilon(\phi-\lambda)+\eta]-\mathrm{erf}[\eta]\} \times \frac{d\sigma_{x=0}(\lambda)}{d\lambda} d\lambda. \quad (16)$$

For the surface heat flux, for arbitrary time variation of the fluid at $x = 0$, application of Duhamel's Theorem yields,

$$\frac{q_w}{k_f \sqrt{(u_\infty/\pi\alpha_f x)}} = -\int_0^{\phi} (\exp\{-[\varepsilon^2(\phi-\lambda)^2+2\eta\varepsilon(\phi-\lambda)]\} + \sqrt{(\pi)}\eta\, e^{\eta^2}\{\mathrm{erf}[\varepsilon(\phi-\lambda)+\eta]-\mathrm{erf}[\eta]\}) \times \frac{d\sigma_{x=0}(\lambda)}{d\lambda} d\lambda. \quad (17)$$

In equations (16) and (17), it is important to note that λ is a dummy variable for τ not for ϕ. The ϕ in the upper limit of the integral is a result of the unit step function $u(\tau-1)$ in the original solution for the step change in fluid temperature.

Linear fluid temperature variation at $x = 0$

The case of the fluid temperature at $x = 0$ varying linearly with time (a ramp in fluid temperature at $x = 0$) is of special interest for two reasons. First, experimental data show that the turbine inlet temperature of a gas turbine engine varies in an approximately linear fashion with time during startup, shutdown, and power level changes (accelerations or decelerations). Second, for many fluid temperature variations with time at $x = 0$, the integrations indicated in equations (16) and (17) cannot be made analytically to yield simple functions. However, for the linear variation of fluid temperature at $x = 0$, the ramp, the integration can be made and yields simple functions. As shown in [25], and others, a fairly arbitrary disturbance function of time can be approximated well by a sequence of ramps or a sequence of ramps and steps. Thus the response to a generalized ramp in fluid temperature at $x = 0$ is sought. Mathematically, this disturbance function is given as,

$$T_{x=0}(t) = A_i + B_i t, \qquad t_i \leqslant t \leqslant t_j. \quad (18)$$

Utilizing equation (15) gives, after rearrangement,

$$\sigma_{x=0}(\lambda) = A_i + \frac{B_i x}{u_\infty}\lambda - T_c.$$

Thus,

$$\frac{d\sigma_{x=0}(\lambda)}{d\lambda} = \frac{B_i x}{u_\infty}. \quad (19)$$

After inserting equation (19) into (16) with the lower limit being

$$\tau_i = u_\infty t_i / x$$

two integrations must be made, one for the case where $\tau_i \leqslant \lambda < \tau_j$, and then a separate one for the case where $\lambda > \tau_j$.

Thus, the temperature response of the wall to the generalized linear temperature variation of the fluid at $x = 0$ is given by

$$\Delta\sigma^{r'}_{w_i} = \frac{B_i x}{u_\infty \varepsilon} e^{\eta^2}\{\varepsilon(\phi-\tau_i)\,\mathrm{erfc}[\eta] + i^1\,\mathrm{erfc}[\varepsilon(\phi-\tau_i)+\eta] - i^1\,\mathrm{erfc}[\eta]\} \quad \text{for} \quad \tau_i \leqslant \phi < \tau_j \quad (20)$$

and by

$$\Delta\sigma^{r}_{w_i} = \frac{B_i x}{u_\infty \varepsilon} e^{\eta^2}\{\varepsilon(\tau_j-\tau_i)\,\mathrm{erfc}[\eta] + i^1\,\mathrm{erfc}[\varepsilon(\phi-\tau_i)+\eta] - i^1\,\mathrm{erfc}[\varepsilon(\phi-\tau_j)+\eta]\} \quad \text{for} \quad \phi \geqslant \tau_j \quad (21)$$

i^1 erfc refers to the first repeated integral of the error function [26]. In equations (20) and (21) the superscript r identifies the disturbance as a linear one, a ramp, while the subscript i refers to the time at which the ramp begins. For completeness, the generalized version of equation (10) as the response to a step which occurs at time t_i is given below.

$$\Delta\sigma^{s}_{w_i} = (T_i^+ - T_i^-)\, e^{\eta^2}\{\mathrm{erf}[\varepsilon(\phi-\tau_i)+\eta]-\mathrm{erf}[\eta]\} \quad \text{for} \quad \phi > \tau_i. \quad (22)$$

Defining for convenience the following heat flux parameter,

$$F = \frac{q_w}{k_f \sqrt{(u_\infty/\pi\alpha_f x)}} \quad (23)$$

the surface heat flux response counterparts of equations (20), (21), and (22) become

$$F_i^{r'} = \frac{B_i x}{u_\infty \varepsilon}\sqrt{(\pi)}\, e^{\eta^2}\{\tfrac{1}{2}[\mathrm{erf}[\eta]-\mathrm{erf}[\varepsilon(\phi-\tau_i)+\eta]] - \eta\varepsilon(\phi-\tau_i)\,\mathrm{erfc}[\eta] + \eta[i^1\,\mathrm{erfc}[\eta] - i^1\,\mathrm{erfc}[\varepsilon(\phi-\tau_i)+\eta]]\} \quad \text{for} \quad \tau_i \leqslant \phi < \tau_j \quad (24)$$

$$F_i^{r} = \frac{B_i x}{u_\infty \varepsilon}\sqrt{(\pi)} \times e^{\eta^2}\{\tfrac{1}{2}[\mathrm{erf}[\varepsilon(\phi-\tau_j)+\eta]-\mathrm{erf}[\varepsilon(\phi-\tau_i)+\eta]] - \eta\varepsilon(\tau_j-\tau_i)\,\mathrm{erfc}[\eta] + \eta[i^1\,\mathrm{erfc}[\varepsilon(\phi-\tau_j)+\eta] - i^1\,\mathrm{erfc}[\varepsilon(\phi-\tau_i)+\eta]]\} \quad \text{for} \quad \phi \geqslant \tau_j \quad (25)$$

$$F_i^{s} = -(T_i^+ - T_i^-)\{\exp\{-[\varepsilon^2(\phi-\tau_i)^2+2\eta\varepsilon(\phi-\tau_i)]\} + \sqrt{(\pi)}\eta\, e^{\eta^2}[\mathrm{erf}[\varepsilon(\phi-\tau_i)+\eta]-\mathrm{erf}[\eta]]\} \quad \text{for} \quad \phi > \tau_i. \quad (26)$$

Equations (20), (21), (22), (24), (25) and (26), when properly combined, give the heat-transfer response functions for any arbitrary combination of step changes and ramps in the fluid temperature at $x = 0$ and these functions can be used to approximate, to any desired accuracy, the heat transfer response to any fluid temperature as a function of time at $x = 0$. As an illustration, a case of interest in gas turbine cooling technology will be utilized. Initially the temperature is T_c, when at $t = 0$ the fluid temperature at $x = 0$ increases linearly with time to a final value of T_0 at time t_1 and thereafter is held at T_0 for all t. Surface temperature and heat flux are desired for this situation. After noting that $\tau_i = 0$, $\tau_j = \tau_1 = u_\infty t_1/x$, and

$$\frac{B_i x}{u_\infty} = \frac{T_0 - T_c}{\tau_1}$$

the solution functions for the temperature of the plate can be written down directly from equations (20) and (21) as

$$\frac{T_w - T_c}{T_0 - T_c} = \frac{e^{\eta^2}}{\varepsilon\tau_1}\{\varepsilon\phi \operatorname{erfc}[\eta] + i^1 \operatorname{erfc}[\varepsilon\phi + \eta] - i^1 \operatorname{erfc}[\eta]\} \quad \text{for} \quad \phi < \tau_1 \quad (27)$$

$$\frac{T_w - T_c}{T_0 - T_c} = \frac{e^{\eta^2}}{\varepsilon\tau_1}\{\varepsilon\tau_1 \operatorname{erfc}[\eta] + i^1 \operatorname{erfc}[\varepsilon\phi + \eta] - i^1 \operatorname{erfc}[\varepsilon(\phi - \tau_1) + \eta]\} \quad \text{for} \quad \phi \geqslant \tau_1. \quad (28)$$

For the flux one uses equations (24) and (25), and then recognizing that some of the functions have already been written down, one obtains, with

$$Q_w = \frac{F_i}{T_c - T_0},$$

$$Q_w = \sqrt{(\pi)}\left\{\frac{e^{\eta^2}}{2\varepsilon\tau_1}\left(\frac{T_w - T_c}{T_0 - T_c}\right)_{\text{step}} + \eta\left(\frac{T_w - T_c}{T_0 - T_c}\right)_{\substack{\text{ramp}\\ \phi < \tau_1}}\right\} \quad \text{for} \quad \phi < \tau_1 \quad (29)$$

$$Q_w = \sqrt{(\pi)}\left\{\frac{e^{\eta^2}}{2\varepsilon\tau_1}[\operatorname{erf}[\varepsilon\phi + \eta] - \operatorname{erf}[\varepsilon(\phi - \tau_1) + \eta]] + \eta\left(\frac{T_w - T_c}{T_0 - T_c}\right)_{\substack{\text{ramp}\\ \phi \geqslant \tau_1}}\right\} \quad \text{for} \quad \phi \geqslant \tau_1 \quad (30)$$

where

$$\left(\frac{T_w - T_c}{T_0 - T_c}\right)_{\text{step}} = e^{\eta^2}\{\operatorname{erf}[\varepsilon\phi + \eta] - \operatorname{erf}[\eta]\} \quad (31)$$

$\left(\frac{T_w - T_c}{T_0 - T_c}\right)_{\substack{\text{ramp}\\ \phi < \tau_1}}$ is equation (27)

and

$\left(\frac{T_w - T_c}{T_0 - T_c}\right)_{\substack{\text{ramp}\\ \phi \geqslant \tau_1}}$ is equation (28).

Quasi-steady analysis

Transient convection problems are often worked using the quasi-steady assumption because of the relative simplicity it affords. Basically the assumption is that the steady state relations are valid at each instant of time as long as instantaneous values of any time dependent quantities are used in the steady state relations. The quasi-steady analysis in [21] employed a time independent heat-transfer coefficient whose x dependency was determined according to that for a constant temperature surface. Actually, however, the heat-transfer coefficient does depend upon time even in the quasi-steady analysis, since the surface temperature variation with x depends upon time. An expression for the quasi-steady surface heat flux in slug flow over a surface with an arbitrary surface temperature distribution in x can be found by specializing the elements of Lighthill's derivation [27] to this simpler case. The result is

$$q_{w,\text{q.s.}} = k_f\sqrt{(u_\infty/\pi\alpha_f)}\int_0^x \frac{1}{\sqrt{(x-\xi)}}\frac{\partial\theta_w}{\partial\xi}\,d\xi.$$

An energy balance on the plate yields

$$\frac{\partial\theta_w}{\partial t} + \frac{1}{r}\sqrt{(u_\infty/\pi\alpha_f)}\int_0^x \frac{1}{\sqrt{(x-\xi)}}\frac{\partial\theta_w}{\partial\xi}\,d\xi = \frac{h_c}{\rho_w C_{p,w} b}(\theta_c - \theta_w). \quad (32)$$

Solving equation (32) by Laplace transforms, the quasi-steady wall temperature distribution and surface heat flux are, for a step change in the fluid temperature at $x = 0$,

$$\frac{\theta_{w,\text{q.s.}}}{\theta_c} = 1 - e^{\eta^2}\{\operatorname{erf}[\varepsilon\tau + \eta] - \operatorname{erf}[\eta]\} \quad (33)$$

$$Q_{w,\text{q.s.}} = \exp-(\varepsilon^2\tau^2 + 2\eta\varepsilon\tau) + \sqrt{(\pi)}\eta\, e^{\eta^2}\{\operatorname{erf}[\varepsilon\tau + \eta] - \operatorname{erf}[\eta]\}. \quad (34)$$

DISCUSSION OF RESULTS

Perhaps the most obvious feature of the heat-transfer response functions, equations (10), (12), (27), and (29), is that the plate at position x does not respond to the fluid temperature change at $x = 0$ and $t = 0$ until the front of fluid at $x = 0$ reaches the position x. This is a consequence of the neglect of axial conduction in both plate and fluid and the slug velocity profile. This phenomenon has been noted by many workers in conjunction with a step change in plate temperature and is discussed in detail by Siegel [2]. One also notes, from inspection of equation (12), that the surface heat flux never attains the infinite or extremely large values (except at $x = 0$) characteristically exhibited by the flux, in the one dimensional conduction regime, for a step change in the plate surface temperature. This is in accordance with the results of Lyman [18] for the stagnation point problem as was explained in the Introduction of the present paper.

Step change in fluid temperature

From equations (10) and (12), response curves for the nondimensional wall temperature and surface heat flux are plotted in Figs. 1 and 2, respectively, for the step change in the fluid temperature. The parameter on these plots is η where $\eta = 0$ is viewed as $h_c = 0$ and

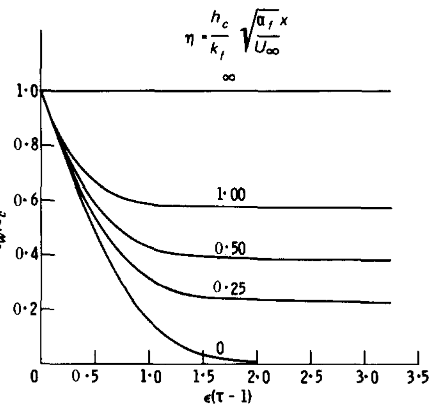

FIG. 1. Wall temperature response to step change in fluid temperature at $x = 0$ with fluid and plate initially at T_c.

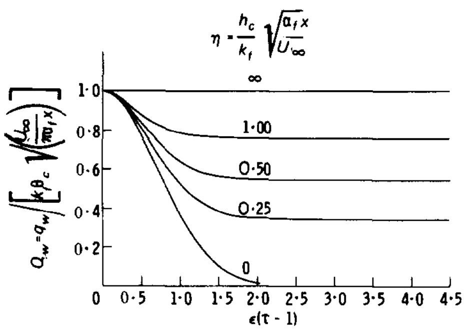

FIG. 2. Surface heat flux response to step change in fluid temperature at $x = 0$ with fluid and plate initially at T_c.

therefore corresponds to an insulated lower plate surface (or a solid uncooled blade or vane in a gas turbine engine), and $\eta = \infty$ is viewed as caused by $h_c = \infty$ which corresponds to the plate always being at T_c. More generally, a rearrangement of the definition of η gives,

$$\eta = \frac{(\sqrt{\pi})h_c}{k_f\sqrt{(u_\infty/\pi\alpha_f x)}}$$

and thus η is a measure of the ratio of the coolant side surface coefficient to the surface coefficient which the top of the plate would experience in a slug flow if the plate was isothermal. This interpretation was the motivation for the values of η used in Figs. 1 and 2 since an η of 1 could correspond to an impingement cooled turbine blade or vane. Study of Figs. 1 and 2 indicate that both Q_w and θ_w/θ_c are within 5 per cent of their eventual steady state values if the time parameter satisfies the following inequality.

$$\varepsilon(\tau - 1) \geqslant 2{\cdot}05 - 1{\cdot}175\eta \quad \text{for} \quad 0{\cdot}25 \leqslant \eta \leqslant 1{\cdot}0. \quad (35)$$

Equation (35) also provides the nondimensional time for $\eta = 0$ if one agrees to use the criterion that both Q_w and θ_w/θ_c be less than 0·02. In addition equation (35) provides the following upper bound on the time to reach 5 per cent of steady state values when $\eta > 1$, namely $\varepsilon(\tau - 1) = 0{\cdot}875$.

Next the nature of the coupling parameter ε and its influence on the surface heat flux are examined. Recalling that, for laminar, steady slug flow over an isothermal plate, the thermal boundary-layer thickness has the form,

$$\delta_t \sim \sqrt{(\alpha_f x/u_\infty)}.$$

Inserting this into the definition of ε and rearranging yields

$$\varepsilon = \frac{\rho_f C_{p,f}\delta_t}{\rho_w C_{p,w} b}.$$

Hence ε is a measure of the ratio of the thermal energy storage capacity per unit length of the boundary-layer fluid to that of the plate material. Hence for $\varepsilon \to 0$, viewed as being caused by a plate of very large thermal energy storage capacity, one would expect, on physical grounds, that the plate remains isothermal at T_c regardless of η, and $Q_w = 1$ for $\tau \geqslant 1$. Indeed this is what equation (12) indicates as does Fig. 3 which plots Q_w vs $\tau - 1$ for a number of different values of ε and of η. For fixed η, it is seen that the larger the value of ε, the shorter the duration of the transient. In the limit as $\varepsilon \to \infty$, viewed as caused by a zero thermal energy storage capacity plate, equation (12) yields

$$Q_w \to (\sqrt{\pi})\eta\, e^{\eta^2}\,\text{erfc}[\eta] \quad \text{for} \quad \tau \geqslant 1$$

that is, the plate responds instantly (at $\tau = 1$) to the new fluid temperature and there is only the simple transient due to the passage of the front of fluid that was at $x = 0$ at $t = 0$. This results from the fact that it is the thermal lag of the plate when interacting with the fluid which causes the major transient in the fluid and in this case the plate has no thermal lag at all. It can also be seen from Fig. 3 that the eventual steady state flux, at any η, is independent of the coupling parameter ε. This is expected physically by virtue of

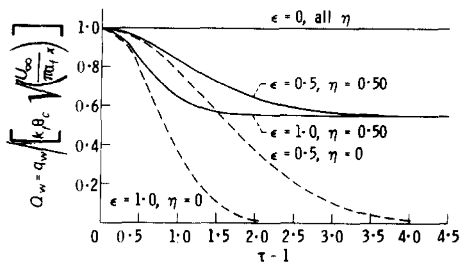

FIG. 3. Influence of coupling parameter ε on surface heat flux response to step change in fluid temperature at $x = 0$.

the fact that the plate temperature was lumped in the y direction and axial conduction was neglected. Hence the plate temperature field is coupled to that of the fluid only through the unsteady term in equation (5) and when $\partial\theta/\partial t \to 0$ as $t \to \infty$, the plate properties no longer appear in the solution functions.

Linear fluid temperature variation

Next is the case where the fluid and plate are initially at T_c and then a linear variation (ramp) of fluid temperature at $x = 0$ occurs until it reaches T_0 at which time, t_1, the fluid at $x = 0$ is held at T_0 for all time thereafter. The solutions for the wall temperature and surface heat flux are given by equations (27), (28) and (29), (30), respectively. It can be seen that an additional parameter, $\varepsilon\tau_1$, appears which was not present in the previously discussed step function solutions. τ_1 is the nondimensional time at which the ramp ends and thus $\varepsilon\tau_1$ is a measure of the steepness of the ramp, the smaller $\varepsilon\tau_1$ is, the steeper the ramp. Some representative wall temperature and surface heat flux distributions are plotted in Figs. 4 through 7 for values of $\varepsilon\tau_1$ indicative of those encountered in turbine blades and vanes. Plotted also on these figures for comparison

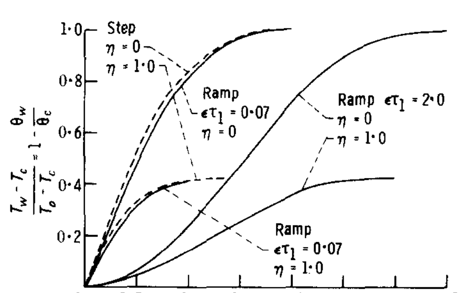

FIG. 4. Wall temperature response to a linear change in fluid temperature at $x = 0$ from T_c to T_0 and comparison with the response to a step change.

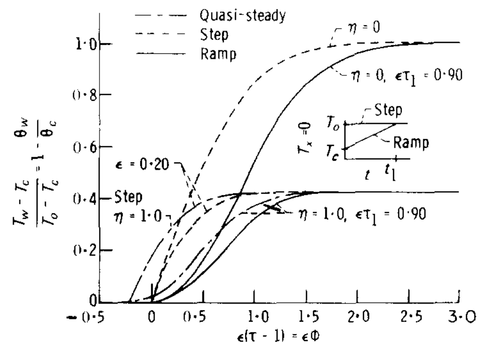

FIG. 5. Wall temperature response to a linear change in fluid temperature at $x = 0$, comparison with response to a step change and with a quasi-steady solution.

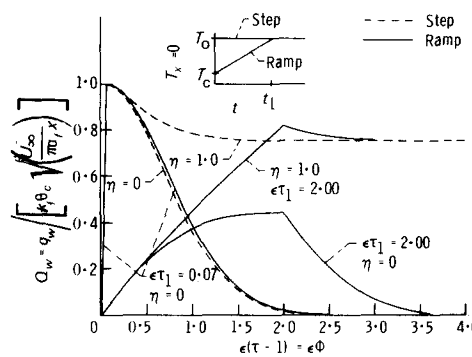

FIG. 6. Surface heat flux response to a linear change in fluid temperature at $x = 0$ and comparison with response to a step change.

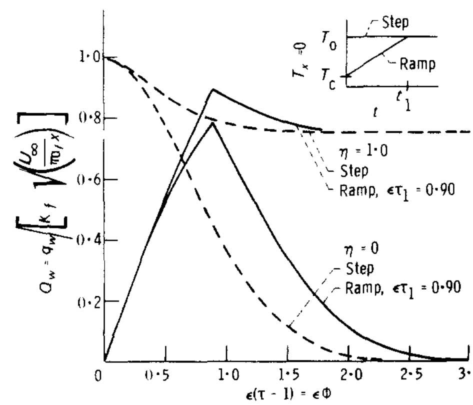

FIG. 7. Surface heat flux response to a linear change in fluid temperature at $x = 0$ and comparison with response to a step change.

purposes are the step function solutions. It is seen from Fig. 4, and even more graphically from Fig. 6, that the response to the steepest ramp shown, $\varepsilon\tau_1 = 0{\cdot}07$, is approaching the response to a step function in fluid temperature as would be expected on physical grounds. However this is not the case for the less steep ramps and hence the step function solution should not be used as an approximation here even though equations (10) and (12) offer simplicity and computational ease relative to the correct equations (27) to (30). The transients in θ_w and Q_w induced by the ramp in fluid temperature are, as expected, less severe than those for a step in fluid temperature since the ramp causes the fluid temperature at $x = 0$ to reach T_0 gradually rather than abruptly as the step requires. Inspection of Figs. 4–7 also leads to the conclusion that both Q_w and θ_w/θ_c for the ramp in fluid temperature are within 0·02 of their steady-state values for all values of η and for $\varepsilon\tau_1 \leqslant 2{\cdot}00$ for

$$\varepsilon(\tau - 1) \geqslant 3{\cdot}5. \tag{36}$$

From a more general standpoint, the curves of Figs. 4–7 are viewed as graphs of the functions on the r.h.s. of equations (27) to (30), functions which are of use for other fluid temperature variations related to the basic ramp. For example, consider the case of steady-state operation with $T_{x=0} = T_1$ where $T_1 \neq T_c$ and then at time $t = 0$ the temperature, $T_{x=0}$, changes linearly with time from T_1 to T_0 in time t_1 and is held at T_0 for all time thereafter. This mode of operation corresponds to a change in power level from one steady state to a final, different, steady state. The heat-transfer response functions are desired. For this case, equations (16) and (17) yield, after noting that some of the required functions are already in hand,

$$\frac{T_w - T_c}{T_0 - T_c} = \left(\frac{T_1 - T_c}{T_0 - T_c}\right) e^{\eta^2} \operatorname{erfc}[\eta] + \left(\frac{T_0 - T_1}{T_0 - T_c}\right) \begin{Bmatrix} f(27) \text{ if } \phi < \tau_1 \\ f(28) \text{ if } \phi \geqslant \tau_1 \end{Bmatrix} \tag{37}$$

$$Q_w = \left(\frac{T_1 - T_c}{T_0 - T_c}\right) (\sqrt{\pi})\eta\, e^{\eta^2} \operatorname{erfc}[\eta] + \left(\frac{T_0 - T_1}{T_0 - T_c}\right) \begin{Bmatrix} f(29) \text{ if } \phi < \tau_1 \\ f(30) \text{ if } \phi \geqslant \tau_1 \end{Bmatrix}. \tag{38}$$

In the above equations, the notation $f(27)$, for instance, means the value of the function given by equation (27). Since, as just discussed, these functions have been graphed in Figs. 4–7, one can use the figures directly in this transient which is more general than the one for which the figures were constructed.

In all of the response curves presented so far, the wall temperature and the surface heat flux variation with time at different x positions has not been shown explicitly because of the fact that the nondimensional quantities, such as ε, τ and η, each depend upon x. So as to show more clearly the time dependency at different x locations and also to make a rough qualitative comparison with the results of Sparrow and DeFarias [21], Figs. 8 and 9 are plotted for the $\eta = 0$ case. Here L is the plate length and a value of $u_\infty t_1/L = 10^4$ has been chosen. In order to facilitate the qualitative comparison with [21], the quantity $\rho_f C_{p,f} \sqrt{(\alpha_f L/u_\infty)}/\rho_w C_{p,w} b$ which is analogous to their a^*, was taken to be 10^{-4}. The inlet temperature in [21] is periodic with angular frequency ω. For the ramp in fluid temperature at $x = 0$ in the present work, where t_1 is the duration of the ramp, the quantity $1/t_1$, is roughly analogous to an angular frequency and $2\rho_w C_{p,w} b \sqrt{(\alpha_f L/u_\infty)}/k_f t_1$, is similar to the quantity b^* of [21] and was taken to be 2·0. Figure 8 shows that at the downstream stations, the wall temperature lags

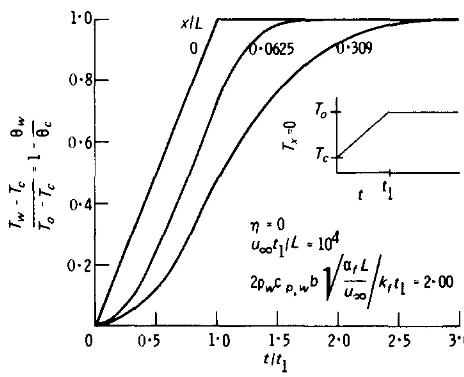

FIG. 8. Wall temperature response to a linear change in fluid temperature at $x = 0$ as a function of x and t.

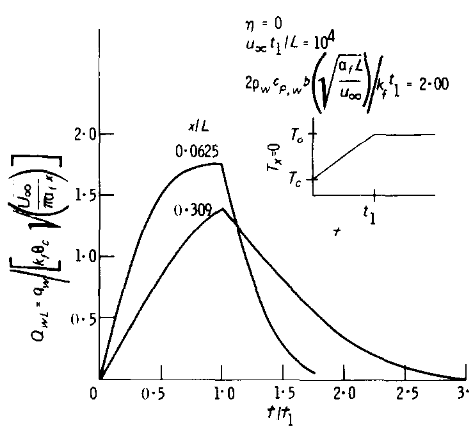

FIG. 9. Surface heat flux response to a linear change in fluid temperature at $x = 0$ as a function of x and t.

that at the upstream stations as would be expected and is in qualitative agreement with [21]. The amplitude of the wall temperature is, of course, not attenuated because, unlike [21], here all of the wall must eventually come to the same temperature T_0. It should also be noted, both in Figs. 8 and 9, that each curve has a different origin, other than $t/t_1 = 0$ (except for the $x/L = 0$ case), due to the propagation time, previously discussed, needed before the front of fluid that was at $x = 0$ at $t = 0$ reaches any other x. This does not show in the figures due to the scale chosen and also because of the value of the parameter $u_\infty t_1/L$.

Quasi-steady results

In accordance with the description given in the Analysis Section, quasi-steady results were found for a number of cases. Two curves of Fig. 5 portray the quasi-steady wall temperature at $\eta = 1{\cdot}0$, for both a step and a ramp in the fluid temperature at $x = 0$ vs time, while Fig. 10 gives quasi-steady surface heat flux as the dashed curves. The approach of the quasi-steady solution to the exact solution as ε gets smaller is shown in Fig. 10. It can also be seen that the quasi-steady solution does not properly predict the nondimensional lag time, $\tau = 1$, which must expire before the plate can receive the information that the fluid temperature at $x = 0$ has changed. This was earlier noted by Lyman [18] in terms of his transit time. By examination of equations (33) and (34), it is seen that if $\varepsilon\tau$, in the quasi-steady solution, is replaced by $\varepsilon(\tau - 1)$, the quasi-steady solution becomes identical to the exact solution. Lyman [18] also found this same general behavior for his quasi-steady stagnation point solution. Hence, in an attack on the more complicated problem involving a nonslug velocity profile, one might, as a first approximation, utilize a quasi-steady solution with the equivalent of τ replaced by $\tau - 1$, as long as axial conduction is still being neglected both in the plate and in the fluid.

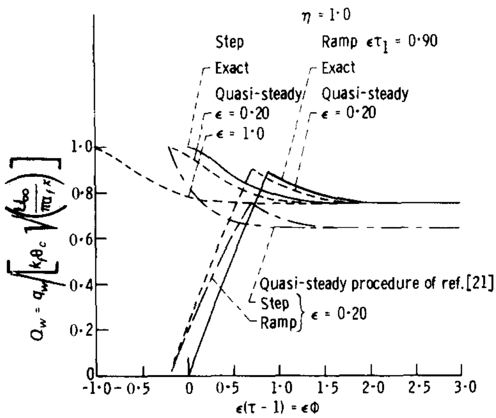

FIG. 10. Comparison between various quasi-steady solutions and exact solutions for surface heat flux response to both a ramp and a step change in fluid temperature at $x = 0$.

Computation shows the error between the quasi-steady solution and the exact solution will generally be less than 10 per cent, for both Q_w and θ_w/θ_c for the step change in fluid temperature and for θ_w/θ_c for the ramp change in fluid temperature, if $\varepsilon \leqslant 0{\cdot}02$. A single general statement cannot be made for Q_w when the fluid temperature varies as the ramp because the error now depends upon whether τ is less or greater than $\tau_1 + 1$ and other factors. Since the exact solution functions presented herein are in easy to use form, this information, concerning the validity of the quasi-steady analysis, can be used as a rough first estimate concerning the applicability of the quasi-steady approach to more complex transient problems of this same general type which involve nonslug velocity profiles, axial conduction in the plate, etc.

In Fig. 10 is also shown, as dashed dot curves, a quasi-steady analysis performed according to the more straightforward procedure of [21] in which a time independent heat-transfer coefficient is used. The x dependency of this heat-transfer coefficient is taken to be that for an isothermal flat plate. For the step change in fluid temperature at $x = 0$, this simpler quasi-steady analysis is more in error than the one discussed previously. Because of the use of the isothermal heat-transfer coefficient, it cannot predict the eventual steady state flux since the plate, even in the steady state, is not isothermal unless $\varepsilon = 0$ or $\eta = \infty$. The comparison is more complicated for the ramp change in fluid temperature. Here the simpler quasi-steady analysis is better at short times and worse at the longer times and again predicts an erroneous steady state.

Magnitude of ε

To gain an appreciation for its magnitude in some situations of possible interest, a few numerical values of the coupling parameter ε were computed. Its value, of course, depends upon x and conditions were generally chosen to give reasonably large values of ε. Thus for an air–Udimet 700 system, $\varepsilon \sim 4 \times 10^{-5}$, for a water–aluminum system, $\varepsilon \sim 0{\cdot}20$, while for a liquid sodium–steel system, $\varepsilon \sim 1{\cdot}00$. The value of ε for the air–Udimet 700 system is an illustration of the small values of ε associated with air combined with almost any solid material for the plate. This agrees qualitatively with the statements in [17], [18], and [21] concerning analogous coupling parameters. Hence, the earlier statement on the validity of the quasi-steady results indicates that the quasi-steady analysis will suffice when air is the heat transfer fluid, may be insufficient when the fluid is water, and certainly can be in serious error when the fluid is a liquid metal.

CONCLUSION

An exact analytical solution has been found for the transient surface heat flux and temperature distribution in the fluid, moving over a plate which is cooled from below, caused by a step change in the fluid temperature at the plate leading edge. The result has been generalized to handle arbitrary fluid temperature variation with time. This is a straightforward procedure, computationally, when one approximates the fluid temperature variation by a sequence of ramps and/or steps for which all the needed response functions are presented herein. Results are also given for the time to reach steady state and for a criterion to determine the validity of a quasi-steady analysis. For air as the fluid, the quasi-steady analysis suffices for practically any plate material and plate thickness.

The solution indicates, for the step change in fluid temperature at $x = 0$ and $t = 0$, the lack of the infinite and very large flux associated with step changes in the wall temperature.

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TRANSFERT THERMIQUE INSTATIONNAIRE ENTRE UN FLUIDE, A TEMPERATURE VARIABLE DANS LE TEMPS, ET UNE PLAQUE: UNE SOLUTION EXACTE

Résumé—En utilisant la transformation de Laplace, on détermine la distribution de température et le flux de chaleur pariétal quand une plaque, refroidie par convection à partir du bas, est léchée par un fluide dont la température au loin varie arbitrairement avec le temps et quand la réponse thermique de la plaque est couplée au fluide par les conditions à l'interface.

La solution d'abord donnée pour une fonction échelon est ensuite généralisée au cas d'une variation arbitraire de la température du fluide en fonction du temps. Une méthode approchée, simple à utiliser, est présentée pour le cas le plus général. Pour comparaison, on donne aussi les résultats du problème quasistationnaire.

INSTATIONÄRER WÄRMEÜBERGANG ZWISCHEN EINEM FLUID MIT ZEITLICH VERÄNDERLICHER TEMPERATUR UND EINER PLATTE: EINE EXAKTE LÖSUNG

Zusammenfassung—Mit Hilfe der Laplace-Transformation wird die zeitliche Temperaturverteilung und der Wärmestrom durch die Oberfläche einer Platte für den Fall untersucht, daß die konvektiv von unten gekühlte Platte von einem Fluid überspült wird, dessen Freistromtemperatur an der Anströmkante regellos mit der Zeit schwankt und daß die thermische Antwort der Platte mit dem Fluid über die Vereinigungsbedingungen an der Zwischenschicht verknüpft ist.

Zuerst wird die Lösung für eine Sprungfunktion ermittelt und diese dann verallgemeinert, um eine regellose zeitliche Temperaturänderung zu behandeln. Eine einfach zu handhabende Näherungsmethode für den allgemeinen Fall wird angegeben, und zum Vergleich werden auch quasi-stationäre Ergebnisse ermittelt.

НЕСТАЦИОНАРНЫЙ ТЕПЛООБМЕН МЕЖДУ ЖИДКОСТЬЮ С ПЕРЕМЕННОЙ ТЕМПЕРАТУРОЙ И ПЛАСТИНОЙ. ТОЧНОЕ РЕШЕНИЕ

Аннотация — С использованием преобразования Лапласа проведен анализ нестационарного распределения температуры и теплового потока на поверхности для случая обтекания конвективно охлаждаемой снизу пластины жидкостью, температура свободного потока которой у передней кромки произвольно изменяется с течением времени, когда тепловая реакция пластины определяется условиями сопряжения на границе раздела. Получено решение для ступенчатой функции температуры, которое затем обобщается для произвольных изменений температуры со временем. Пример использования приближенного метода представлен для наиболее общего случая. Для сравнения получены также квазистационарные результаты.